Gradient Projection Algorithms for Arbitrary Rotation Criteria in Factor Analysis

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The factor analysis model

$$X = \Lambda\xi + U$$

where

X are multivariate normal random vectors

Λ are factor loadings

ξ are common factors

U are error of measurement

Exploratory factor analysis model:
no prior values for factor loadings

Applied in psychology, economics, chemistry, survey data, etc.

The problem

A well known issue in exploratory factor analysis is that the solution is unique only up to a rotation of the factor loadings

In the orthogonal case

$$\Lambda = AT$$

where A is an initial solution, and is T any orthogonal matrix.

In the oblique case

$$\Lambda = A(T')^{-1}$$

where T is any normal matrix.

The rotation problem is to find T such that Λ is simple and looks good. This is usually done by optimizing some criterion.

Criteria

Let Q be a smooth criterion function that measures the complexity of Λ.

Orthogonal: Minimize

$$f(T) = Q(\Lambda) = Q(AT)$$

over all orthogonal matrices.

Oblique: Minimize

$$f(T) = Q(\Lambda) = Q(A(T')^{-1})$$

over all normal matrices.

Gradient projection for factor analysis

0. Choose initial T.

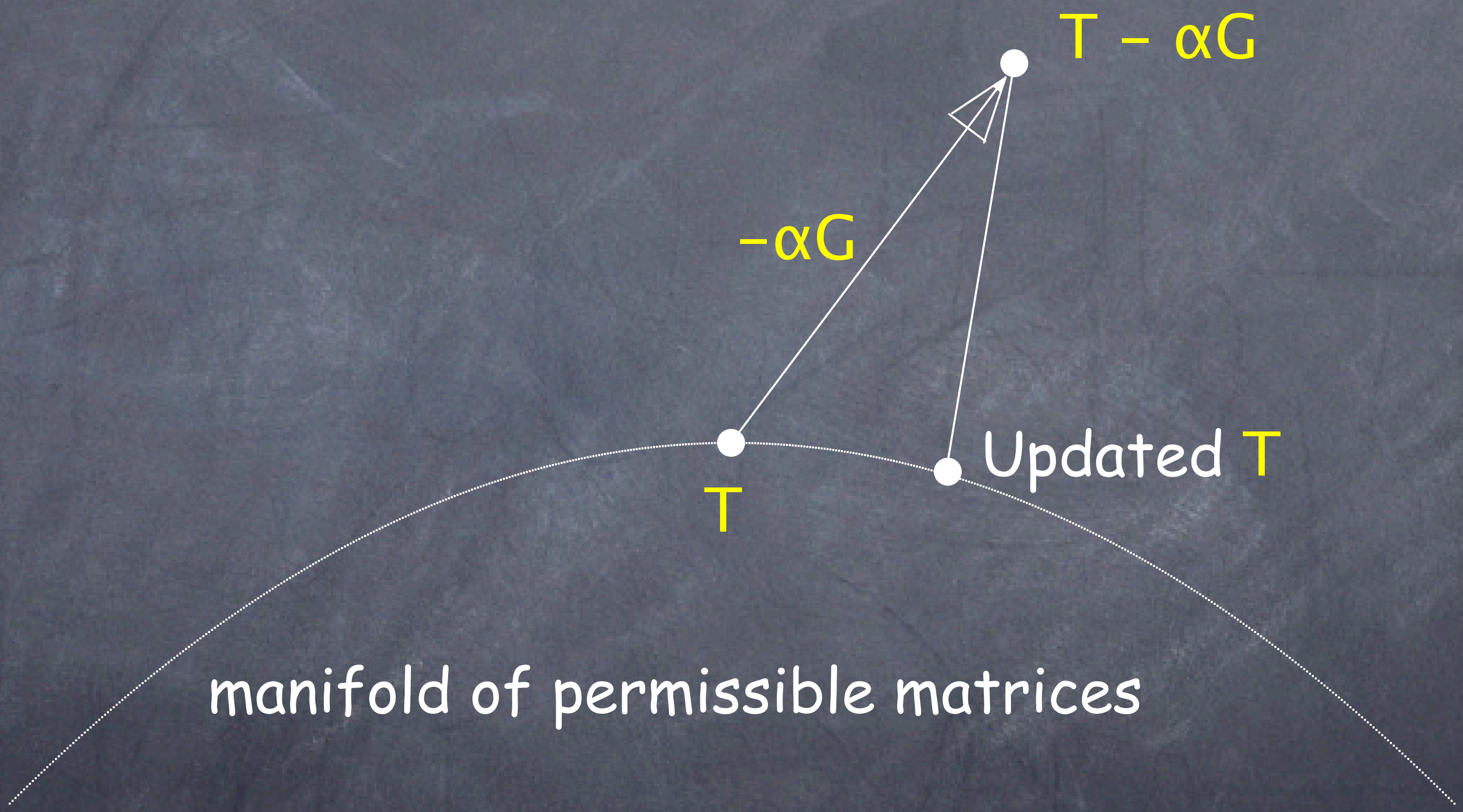
1. Compute $G = df/dT$, gradient G of f at T, the matrix of partial derivatives.

2. Replace T by its projection of $T - \alpha G$ onto the manifold of permissible matrices.

3. Goto step 1 or stop.

Function values are **minimized**

The algorithm visualized



Algorithm properties

Can be shown to decrease value of f at every iteration, with appropriate α. A partial step modification is used to guarantee this.

How to compute G

In step 1, $G = df/dT$ is needed

Let $G_q = dQ/d\Lambda$ is gradient of Q at Λ. Because

$G = A'G_q$ (orthogonal)

$G = -(\Lambda' G_q T^{-1})'$ (oblique)

all that required to compute G is G_q.

Mathematical definitions

Frobenius norm: $(X, Y) = tr(X'Y)$
Elementwise product: $X\ Y$ and $X^2 = X\ X$

Example: Quartimax (orthogonal only)

$$Q(\Lambda) = -(1/4) \sum_i \sum_r \lambda_{ir}^4 = -(\Lambda^2, \Lambda^2)/4$$

Note: Minus because of minimization.

$$dQ = -(\Lambda\ d\Lambda, \Lambda^2)/2 - (\Lambda^2, \Lambda\ d\Lambda)/2$$
$$= -(\Lambda\ d\Lambda, \Lambda^2) = -(\Lambda^3, d\Lambda)$$

Thus, $G_q = -\Lambda^3$

Currently implemented

Orthogonal

Crawford-Ferguson (quartimax, varimax, equamax, parsimax, factor parsimony), orthomax (quartimax, varimax, equamax), invariant pattern simplicity, minimum entropy, oblimax, simplimax, tandem criterion I and II, target rotation.

Oblique

Oblimin (quartimin, biquartimin), Crawford-Ferguson, geomin, invariant pattern simplicity, simplimax, target rotation.

How to obtain the algorithms?

Routines can be downloaded free of charge from
http://www.stat.ucla.edu/research/gpa

SAS PROC IML
SPSS matrix language
Splus
R
Matlab
(Stata)

Alternatives

Pairwise rotation. Implemented in
CEFA (Browne et al., 1998)
http://quantrm2.psy.ohio-state.edu/browne

Browne (2001). An overview of analytic rotation procedures in exploratory factor analysis. Multivariate Behavioral Research, 36, 111-150.

Questions?

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